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FINITE ELEMENT CALCULATIONS FOR AERODYNAMIC COEFFICIENTS  
OF A 3-DIMENSIONAL BODY IN SUBSONIC FLOW USING GREEN'S  
FUNCTION METHOD

Mitsunori Yanagizawa and Kazuo Kikuchi

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## Report Documentation Page

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### ABSTRACT

In recent years, finite element calculation schemes for the evaluation of aerodynamic loads have been developed using the Green's function method. In these schemes, a set of linear algebraic equations is obtained by the discretization of the Fredholm integral equation of second kind, and is solved to yield values of velocity potential on the surface. The pressure on the surface is obtained by evaluating the derivatives of the velocity potential along the stream line. In this report, we propose an accurate method for evaluating the derivatives along circular paths on the surface. Calculations are made on various practical configurations such as wing-body combinations, tandem wings, wings with the dihedral angles at sideslip, ground effects, interference between a sphere and wind-tunnel etc. Comparisons with experiments show good agreement.

### 1. Introduction

Accurate evaluation of aerodynamic loads for the complex shapes of aircraft and flying objects such as lighter-than-air systems or shuttles by solving the potential theory is an important subject. Especially with the trend toward larger aircraft, the structure has more pliability; thus the aerodynamic loads of the fuselage have to be evaluated by considering geometrical change of the wing and the wide body. For static loads, there are problems such as divergence of wing; thus it is especially necessary for this analysis.

The authors analyzed the stream around a 3-dimensional object which creates lift with the finite element method that can be applied to sub-sonic flow and made comparisons with the measured values of the experiments of various models i.e., calculations were made for sphere, ellipsoid of revolution, rectangular wing, wing with wing tip tank, wing with wing tip fence, mid-wing, low-wing, ground effects, interference between an object and the wall of the wind-tunnel.

The stream around a 3-dimensional object in the uniform potential flow can be expressed with an integral equation by the surface singularity method.

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\*\*\*Numbers in margin indicate pagination  
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The surface singularity method expresses a singular solution of the Laplace equation which is a control equation of the potential flow and superposition of distribution of a source or doublet that meets the conditions of infinite-points on the surface of the object instead of solving the Laplace equation with the boundary conditions at the surface of the object and at infinite-point. Then the intensity of the source or doublet is determined so that it meets the boundary conditions on the surface of the object. Therefore, for simplicity it can be converted to the problem of the surface of the object in 2-dimensional space instead of solving for problem of the entire flow in 3-dimensional space. /2

The person who first analyzed the problem of 3-dimensional outer flow with such a finite element method is Hees and Smith (Reference 1). They analyzed for the flow around objects which do not create lift by distributing only sources on the surface of the object. Rubbert and Saaris (Reference 2) made a formulation by locating a source on the surface of the object and distributing doublets to the trailing vortex plane and along the camber of the wing.

Heckmann and Vitte (Reference 3) used a source for expression of effects of thickness, and vortex was used for effect of lift. They then formulated the object which creates lift.

On the other hand, Woodward (Reference 4) and Miranda (Reference 5) had solved the problem by expressing the thickness of the object and effects of lift with only vortices. For expression of the wing thickness with only vortices, the vortices were located in two upper and lower rows equally spaced so that the effects agree with the boundary condition.

In this report, the Green's function method which is a kind of surface singularity method that L. Morino (Reference 6) performed is discussed. This method is derived by applying the Green's function to the Laplace equation and the boundary condition has already been substituted in the integral equation. But the solution is the disturbance velocity potential, so the velocity and pressure have to be calculated by differentiating along the surface of the object.

The differential calculation method for the surface is especially discussed in this report and also the comparison with the experiments of various models is discussed. Refer to the individual report for the detail.

## 2. Symbols

|               |                                                        |
|---------------|--------------------------------------------------------|
| $A$           | : Aspect ratio                                         |
| $a$           | : Position vector at point $P_{ij}$                    |
| $b$           | : Vector from point $P_{i,j}$ to point $P_{i+1,j}$     |
| $B_{hk}$      | : Aerodynamic influence coefficient                    |
| $C$           | : Vector from point $P_{i,j}$ to point $P_{i-1,j}$     |
| $C_{hk}$      | : Aerodynamic influence coefficient                    |
| $C_p$         | : Pressure coefficient                                 |
| $C_F$         | : Vector coefficient of aerodynamic load               |
| $C_M$         | : Moment coefficient                                   |
| $C_L$         | : Lift coefficient                                     |
| $C_W$         | : Trailing vortex path                                 |
| $n$           | : Unit normal vector                                   |
| $n_n$         | : Unit normal vector of circle                         |
| $n_r$         | : Unit vector in radial direction of circle            |
| $NC$          | : Number of panels chordwise of wing                   |
| $NS$          | : Number of panels spanwise of wing                    |
| $O$           | : Origin                                               |
| $O_c$         | : Center of circle through three points                |
| $P$           | : Arbitrary point on surface of object                 |
| $Q$           | : Arbitrary point on surface of object                 |
| $Q'$          | : Arbitrary point on surface of trailing vortex        |
| $R$           | : Distance from point $P$ to point $Q$                 |
| $\mathbf{R}$  | : Position vector of panel center                      |
| $R_R$         | : Position vector of reference rotation axis           |
| $r_o$         | : Position vector of center of circle                  |
| $s$           | : Distance from point $P_{i,j}$                        |
| $S_A$         | : Surface of object                                    |
| $S_W$         | : Trailing vortex plane                                |
| $S_R$         | : Reference area                                       |
| $T_R$         | : Reference length                                     |
| $t_R$         | : Unit vector of reference rotation axis               |
| $U$           | : Velocity vector                                      |
| $W_{hk}$      | : Aerodynamic influence coefficient by trailing vortex |
| $\alpha$      | : Angle of attack                                      |
| $\beta$       | : Side slip angle                                      |
| $\Gamma$      | : Dihedral                                             |
| $\delta_{hk}$ | : Kronecker's delta                                    |

$\lambda$  : Vector in tangent direction of circle  
 $\Sigma_k$  : Element of No. k of object surface  
 $\Sigma_k'$  : Element of No. k of trailing vortex plane  
 $\phi$  : Velocity potencial  
 $\varphi$  : Disturbance velocity potential

### 3. Formation for continuous flow

For the potential flow around a 3-dimensional object in uniform flow, the Laplace equation for disturbance velocity potential is

13

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad \dots\dots (1)$$

As the boundary conditions, the following three conditions were used.

B.C.1 : The stream flows along the surface  $S_A$  of object.

$$\frac{\partial \varphi}{\partial n} = -U_\infty \cdot n \quad \dots\dots (2)$$

B.C.2 : The difference  $\Delta \varphi$  of potential at the upper and lower planes  $S_W$  of the trailing vortex is constant along the stream line.

$$\Delta \varphi = \varphi_+ - \varphi_- = \text{const} \quad \dots\dots (3)$$

B.C.3 : The stream becomes an uniform flow at infinite-point\*

According to Morino's method, when Green's formula is used for the surface of the object which consists of the surface of object  $S_A$  and the surface of the trailing vortex, the following equation can be obtained.

$$4\pi\varphi(P) = \iint_S \varphi(Q) \frac{\partial}{\partial n(Q)} \left[ \frac{1}{R(P,Q)} \right] ds - \iint_{S'} \frac{1}{R(P,Q)} \frac{\partial \varphi(Q)}{\partial n(Q)} ds \quad \dots\dots (4)$$

In this equation, Point Q (x,y,z) is on S and P (x<sub>\*</sub>, y<sub>\*</sub>, z<sub>\*</sub>) is inside of the flow. Thus the distance R between P and Q is

$$R(P,Q) = \sqrt{(x-x_*)^2 + (y-y_*)^2 + (z-z_*)^2} \quad \dots\dots (5)$$

Now, when point P comes close to the surface of the object and when equations (2) and (3) are substituted in equation (4),

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\* The surface  $S_W$  of the trailing vortex is not set up to infinite point. In this calculation, 100 times the reference length is used for the surface of the trailing vortex.

$$\begin{aligned}
2\pi\varphi(P) - \oint_{S_A} \varphi(Q) \frac{\partial}{\partial n(Q)} \left[ \frac{1}{R(P,Q)} \right] ds \\
- \iint_{S_W} \varphi(Q') \frac{\partial}{\partial n(Q')} \left[ \frac{1}{R(P,Q')} \right] ds \\
= \oint_{S_A} U_\infty \cdot n(Q) \frac{1}{R(P,Q)} ds \quad \dots\dots\dots (6)
\end{aligned}$$

where  $\oint$  is the integral of the principal value of Cauchy and point  $Q'$  is the point on the plane of the trailing vortex. Fig.1.

Equation (6) is the Fredholm integral equation of the second kind for  $\varphi$ . Thus the integral equation is solved for the disturbance velocity potential  $\varphi$ . As one method, the surface of the object is broken into panels and  $\varphi$  is assumed constant inside of a panel and it is discretized.

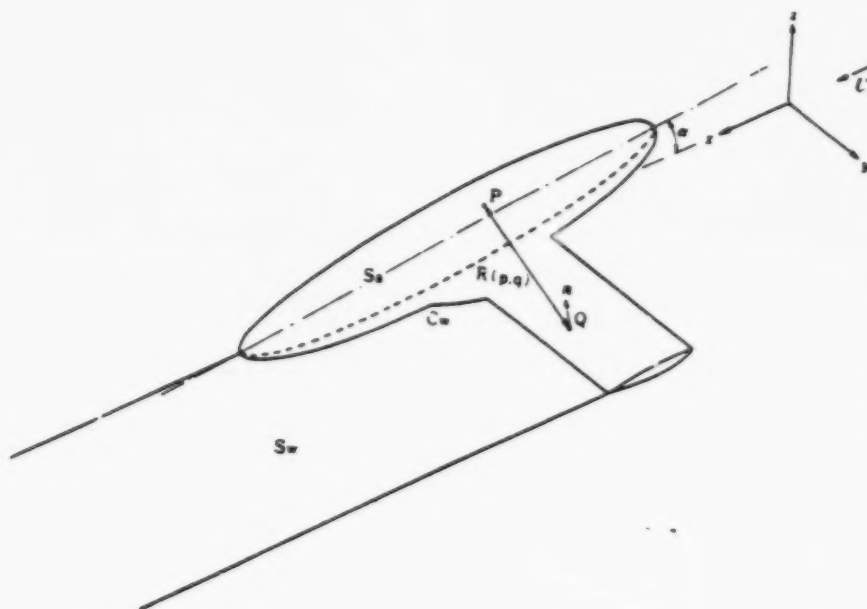


Fig. 1 Formation of a wing to body joint model

#### 4. Discretization by the finite element method (Paneling)

/4

The integral equation (6) is discretized with the following steps.

(1) The surface of the object and the plane of the trailing vortex are divided into rectangular elements  $\Sigma_k, \Sigma_{k'}$ .

(2) The variables  $\varphi, \Delta\varphi$  and  $n$  of each element are represented by the values at the center of the element and the distribution in the element is assumed constant.

(3) The value of  $\Delta\varphi$  is approximated by the difference of the values of  $\varphi$  of the upper and lower elements, where  $C_w$  is one side of the elements, of the surface of the object.

From the above method, the disturbance velocity potential  $\varphi$  on the surface  $S_A$  of the object is discretized to the degree of freedom of the number of divisions of elements with the following simultaneous linear equation. Fig.2.

$$(\delta_{hk} C_{hk} - W_{hk}) \{\varphi_k\} = \{B_{hk}\} \{U_\infty \cdot n_k\} \quad \dots\dots\dots (7)$$

where  $\delta_{hk}$  is Kronecker's delta and the influence coefficients  $C_{hk}, B_{hk}$  and  $W_{hk}$  are

$$\begin{aligned} C_{hk} &= \left[ \frac{1}{2\pi} \oint_{\Sigma_k} \frac{\partial}{\partial n} \left( \frac{1}{R} \right) ds \right]_{r=r_k} \\ B_{hk} &= \left[ \frac{1}{2\pi} \oint_{\Sigma_k} \frac{1}{R} ds \right]_{r=r_k} \\ W_{hk} &= \left[ \frac{\pm 1}{2\pi} \iint_{\Sigma_{k'}} \frac{\partial}{\partial n} \left( \frac{1}{R} \right) ds \right]_{r=r_k} \end{aligned}$$

(Supplement 1)

where  $\oint$  is the principal value of Cauchy.  $W_{hk}$  is zero in the case that element  $\Sigma_k$  is not next to  $C_w$  and the upper (lower) surface next to  $C_w$  is  $+(-)$ .

Supplement 3.

Equation (7) is a simultaneous linear equation taking the type of  $[A]\{\varphi\} = \{B\}$ , the matrix  $[A]$  is a matrix of aerodynamic influence coefficient and  $\{B\}$  is a column matrix calculated from the boundary condition. The column matrix  $\{\varphi\}$  of disturbance velocity potential is unknown.

$$\{\varphi\} = (A)^{-1} \{B\} \quad \dots\dots\dots (8)$$

$\{\varphi\}$  is solved by multiplying  $\{B\}$  and the inverse matrix of  $[A]$ . When the total number of panels of the surface of the object is  $N$ , the number of memories required for the matrix  $[A]$  is  $N^2$  and this requires the maximum memory capacity in this program. In the case of complex models, a solution program of a simultaneous linear equation using an external memory device is used. Supplement 5.

The disturbance velocity potential  $\varphi$  obtained from the above method is differentiated along the surface of the object; then the velocity and the pressure are calculated.

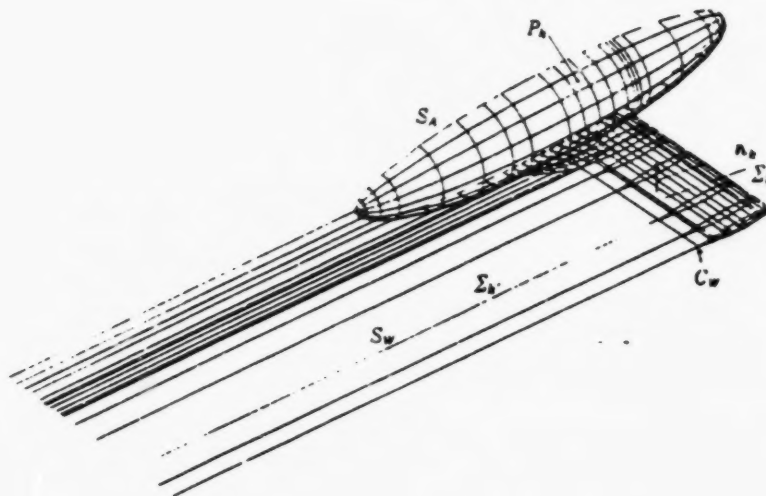


Fig. 2 Discretized by paneling

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## 5. Differentiation of velocity potential

The potential  $\Phi$  of three panels next to each other is differentiated with the assumption that the change can be expressed with a quadratic equation of the distance  $s$  of the surface.

$$\Phi = U_{\infty}(x + \varphi) = as^2 + bs + c \quad \dots\dots\dots (9)$$

where  $a$ ,  $b$  and  $c$  are the coefficients of the quadratic curve and are obtained from /5 the calculation of the velocity potential of the three points. Thus the differentiations of the  $s_1$  and  $s_2$  direction are

$$\frac{d\Phi}{ds_1} = \Phi_{s_1} = 2as_1 + b_1 \quad \dots\dots\dots 10$$

$$\frac{d\Phi}{ds_2} = \Phi_{s_2} = 2as_2 + b_2 \quad \dots\dots\dots 11$$

Next, the tangent unit vectors in the  $s_1$  and  $s_2$  direction at point  $P$  are obtained by the following. In the case that the three points next to each other are located on a straight line, the tangent unit vector is

$$\lambda = \frac{b}{b_1} \quad \dots\dots\dots 12$$

In the case that the three points are not on a straight line, a circle which passes through the three points is considered. Here, the position vector at the center of the circle is  $r_0$ , the position vector at point  $P$  is  $a$ , the vector between  $P_{i,j}$  and  $P_{i-1,j}$  is  $c$ , the vector between  $P_{i,j}$  and  $P_{i+1,j}$  is  $b$ . Then  $r_0$  can be written as follows with the vectors  $a, b, c$ . Fig.3.

$$r_0 = a + s_c b + t_c c \quad (13)$$

where

$$s_c = \frac{(b^2 \cdot b \cdot c) \cdot c^2}{2((b^2 \cdot c^2 - (b \cdot c)^2))}$$

$$t_c = \frac{(b \cdot c - c^2) \cdot b^2}{2((b \cdot c)^2 - b^2 \cdot c^2)}$$

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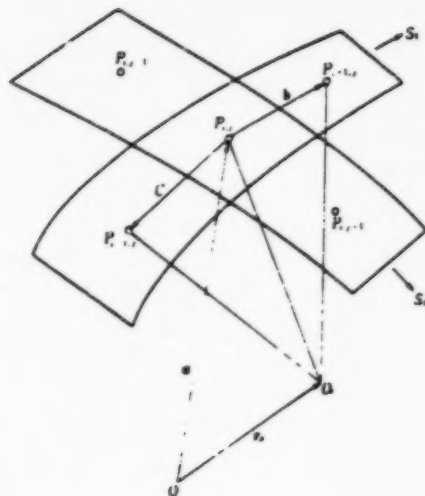


Fig. 3. Differential method by a circle which passes through 3 points

Thus the tangent unit vector  $\lambda$  at point  $P_{i,j}$  is written as follows with  $r_0$ ,  $a$ ,  $b$ ,  $c$ ,  $n_n$  and  $n_r$ .

$$\lambda = n_n \times n_r \quad \dots\dots\dots 04$$

where  $n_n$  is the normal vector to the plane of the three points and  $n_r$  is the vector in the radial direction from the center of the circle toward point P.

$$n_n = \frac{b \times c}{|b \times c|} \quad \dots\dots\dots 05$$

$$n_r = \frac{a - r_0}{|a - r_0|} \quad \dots\dots\dots 06$$

Finally, the tangent unit vectors  $\lambda_1$  and  $\lambda_2$  in directions  $S_1$  and  $S_2$  are

$$\lambda_1 = \frac{\partial_1}{|\partial_1|} \quad \lambda_2 = \frac{\partial_2}{|\partial_2|}$$

in the case that the three points are on a straight line.

$$\lambda_1 = n_{n1} \times n_{r1} \quad \lambda_2 = n_{n2} \times n_{r2}$$

in the case that the three points are on a curve.

The differentiation in the  $e_1$  and  $e_2$  direction can be written as follows when  $e_1$  is in the  $S_1$  direction and  $e_2$  is in the direction normal to  $S_2$  and is tangent to panel P.

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$$\phi_{,1} = \frac{d\phi}{ds_1} = \frac{d\phi}{ds_1} \dots\dots\dots 07$$

$$\phi_{,2} = \frac{d\phi}{ds_2} = \frac{1}{1 - (\lambda_1 \cdot \lambda_2)^2} \left\{ \frac{d\phi}{ds_2} - (\lambda_1 \cdot \lambda_2) \frac{d\phi}{ds_1} \right\} \dots\dots\dots 08$$

Therefore, the pressure coefficient and velocity can be obtained as follows.

$$C_p = 1 - \frac{(\phi_{,1}^2 + \phi_{,2}^2)}{U_\infty^2} \dots\dots\dots 09$$

$$v = \phi_{,1} \cdot \lambda_1 + \phi_{,2} \cdot \lambda_2 - (\lambda_1 \cdot \lambda_2) \cdot \lambda_1 \dots\dots\dots 20$$

It is difficult to make paneling with a grid of the base for a three-dimensional object and to differentiate along the surface, but with this method it can be done easily.

Generally, when the surface of the object is paneled, a large boundary is made at the junction such as the trailing edge of wing or wing to fuselage. The extensive boundary occurs not only because of the geometrical boundary but also because of changing the number of divisions on the continuous surface. Thus at first the surface of a three dimensional object is divided into large blocks, then each block is divided into the same numbers longitudinally and laterally. For instance, a low wing aircraft is extensively divided into 9 blocks. Fig. 4.

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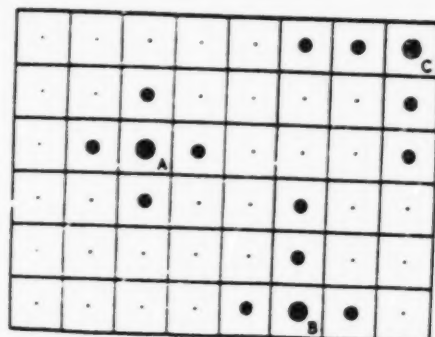


Fig. 4 Differentiation method along the surface of a three dimensional object.

The surface surrounded by the extensive boundary is differentiated for each block. As shown in Fig. 4, in the case that the point to be obtained is in the boundary, i.e., for the differential of point A, the calculation is performed by using the potential  $\Phi$  of the 5 points indicated with a  $\odot$  and a  $\bullet$  in the figure due to existence of the surrounding panels, left, right, front and back.

The differential at point B which contacts the extensive boundary is obtained by using  $\Phi$  of the adjacent points left and right and two points in front of point B as shown in Fig. 4. In the case that the point is located on the corner like point C, the program uses  $\Phi$  of two points on the left hand and two points at the back.

Therefore, the differentiated values are obtained in the order of input and the value at the center of the panel is obtained with the velocity distribution and the pressure distribution. Since the pressure coefficients of all of the panels can be obtained without missing any panels if the paneling division method is planned at first, the lift coefficient and the moment coefficients of the entire object can be computed.

When these aerodynamic coefficients are computed, coincidence of area of the element of the base and the element which is divided at first is very convenient.

#### 6. Calculation of aerodynamic coefficients

Since the pressure at the center of each panel is obtained, the aerodynamic coefficients can be obtained as follows by multiplying the area of each element and the normal vector. The elements are quadrilaterals so the area was obtained by using Helon's equation which gives the area of a triangle twice.

$$C_p = \frac{1}{S_r} \iint_{s_A} C_p n ds = \frac{1}{S_r} \sum_{i=1}^4 C_{p_i} n_i ds_i \quad \dots\dots\dots 20$$

$$\begin{aligned} C_u &= \frac{1}{S_r T_R} t_R \cdot \iint_{s_A} (R - R_0) \times n C_p ds \\ &= \frac{1}{T_R} t_R \cdot (C_x + (R_0 - R_r) \times C_r) \end{aligned} \quad \dots\dots\dots 22$$

where

$$C_r = \frac{1}{S_r} \iint_{s_A} (R - R_0) \times n C_p ds$$

and  $S_r$  is the reference area,  $T_R$  is the reference length,  $t_R$  is the unit vector in the

rotation direction of the reference point,  $R_R$  is the potential vector of the reference point,  $R_0$  is the origin,  $n$  is the normal vector of each panel and  $\Delta S_i$  is the area of the element.

## 7. Calculation results

For investigation of the calculation results of this method, the calculation results and the analysis solutions which are known for sphere and ellipsoid of revolution (reference 8) were compared. Also, the calculations for rectangular wing, wing with wing tip tank, wing with wing tip fence, mid-wing, low-wing, ground effects and interference between an object and of wind-tunnel were performed.

### 7.1 Sphere

Fig. 5 shows the calculation results of the pressure distribution of the surface of a sphere. The total number of panels is 320 and the number of divisions in the stream direction NC is 20 and the number of divisions in the circumferential direction NS is 16. In the case of a sphere, the calculation is performed with good accuracy in the entire zone and the error was within 1.6%. Table 1. This is from the results of the method which, when velocity is obtained from the velocity potential, it is differentiated with the circle that passes through three points.

### 7.2 Ellipsoid of revolution

An ellipsoid of revolution which has a large ratio of length/thickness was selected. The ratio of length  $l$  and the maximum diameter  $d$ ,  $l/d$ , of the object is 5. The calculation was performed with the conditions where the object is set with  $0^\circ$  of angle of attack against the uniform flow. In this case, the trailing vortex was not used because the lift is 0. Fig. 6 shows the pressure distribution obtained from using the total number of panels of 224 with NC = 14 and NS = 16.

The calculation results agree with the analyzed solutions in the zone of 20% to 80% of the object length with 1.0% of error. The forward and afterward FEM values where the slope against the uniform flow direction is large are slightly large. However, even in the places where the pressure change is large, error can be reduced by increasing the number of panels in the flow direction. /7

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| $x$    | $C_p$ (exact) | $C_p$ (FEM) | 1- 误差( $C_{pe} - C_{pf}$ ) | 2- 相对误差<br>$ C_{pe} - C_{pf}  /  C_{pmax} $ |
|--------|---------------|-------------|----------------------------|---------------------------------------------|
| 0.0625 | -1.241        | -1.231      | -0.01                      | 0.008                                       |
| 0.1875 | -1.171        | -1.161      | -0.01                      | 0.008                                       |
| 0.3125 | -1.030        | -1.022      | -0.008                     | 0.0064                                      |
| 0.4375 | -0.8193       | -0.8124     | -0.0069                    | 0.0055                                      |
| 0.5625 | -0.5381       | -0.5305     | -0.0076                    | 0.0061                                      |
| 0.6875 | -0.1865       | -0.1743     | -0.0122                    | 0.0098                                      |
| 0.8125 | 0.2354        | 0.2535      | -0.0181                    | 0.0145                                      |
| 0.9    | 0.5725        | 0.5781      | -0.0056                    | 0.0045                                      |
| 0.95   | 0.7806        | 0.7975      | -0.0169                    | 0.0135                                      |
| 0.9875 | 0.9441        | 0.9649      | -0.0208                    | 0.0166                                      |

Table 1. Comparison of pressure distribution of sphere obtained from FEM and analytical solution

1- Error ( $C_{pe} - C_{pf}$ )

2- Relative error

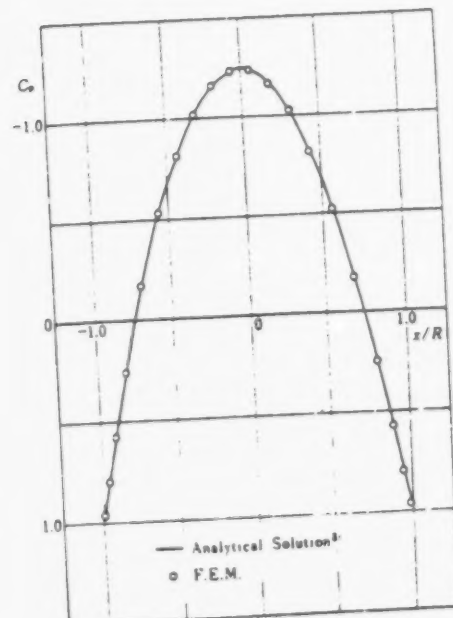


Fig. 5 Comparison of pressure distribution of a sphere obtained from analysis and from FEM.

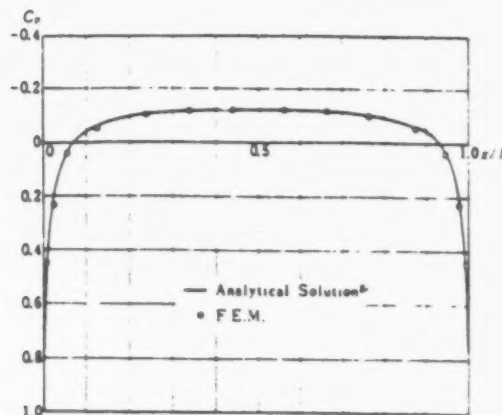


Fig. 6 Pressure distribution of ellipsoid of revolution ( $\alpha = 0^\circ$ )

### 7.3 Rectangular wing

Rectangular wings of aerofoil NACA65A010 and aspect ratio of 5 and 1 were selected for this experiment. Both wings were set for  $4^\circ$  of angle of attack and the distribution of local lift coefficient is shown in Fig. 7. The number of total panels of these models is 224. The local lift coefficient  $C_L(y)$  of the wing of aspect ratio 1 is less than 1/2 of that of aspect ratio 5. In both models, the distribution of  $C_L(y)$  is elliptical. Table 2 shows the comparison of the lift slope of the entire wing obtained by the FEM and obtained by the vortex theory of the past.

$$C_{L\alpha} = \frac{2\pi A}{2 + \sqrt{A^2 + 4}} \quad \dots\dots\dots (2)$$

where A is the aspect ratio of the wing.

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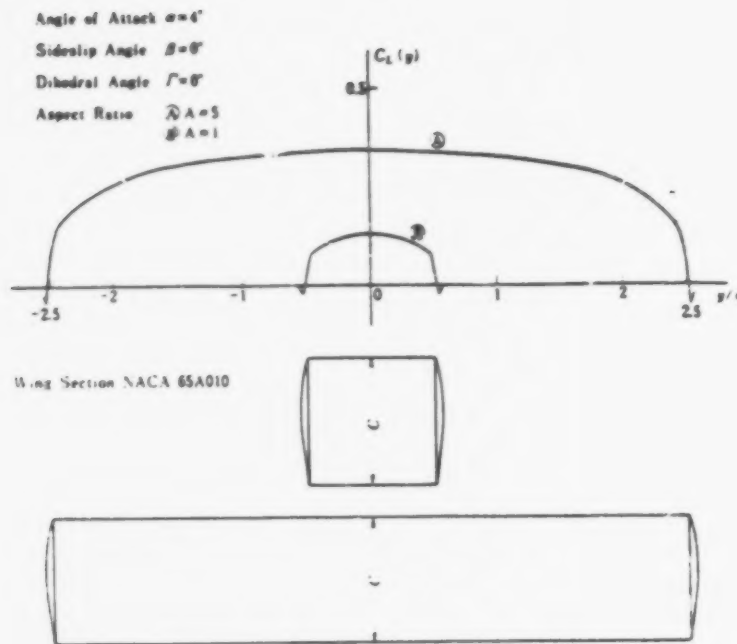


Fig. 7 Difference of local lift distribution by different aspect ratios of the rectangular wing.

| $A \backslash C_{L\alpha}$ | $C_{L\alpha} \text{ (FEM)}$ | $C_{L\alpha} \text{ (Vortex)}^*$ |
|----------------------------|-----------------------------|----------------------------------|
| 1                          | 1.598                       | 1.483                            |
| 5                          | 3.931                       | 4.254                            |

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Table 2 Comparison of lift slope by different aspect ratios A of wing

1- \* Calculated with equation(23)

#### 7.4 Wing with side slip

The velocity distribution and the pressure distribution of the wing with aspect ratio 5 mentioned in paragraph 7.3 with  $10^\circ$  of dihedral  $\Gamma$  is shown in Fig. 8a and b for side slip condition. Fig. 9 shows the comparison of the FEM values and the experiment values of the pressure distribution at both wing tip areas at the conditions of the wing with  $10^\circ$  of dihedral  $\Gamma$ ,  $6^\circ$  of angle of attack  $\alpha$ , and  $10^\circ$  of side slip angle  $\beta$ .

The number of panel divisions of this model is  $NC=10$  and  $NS=7$  and the total number of panels is 360 including the wing tip fairing.

In the calculation of the wing under side slip conditions, the symmetric property of the axis of the aircraft can not be used. Each influence coefficient for left and right panels has to be obtained; thus 4 times the storage capacity of the computer is required compared with the condition without side slip. Supplement 2.

The problem for side slip is the necessity of emission of the trailing vortex from the wing tip (Reference 9). In this calculation, the vortices were emitted from the trailing edge and the interface of wing and wing tip fairing. Fig 10 shows the local lift distribution at the condition of  $\gamma=0^\circ$ ,  $10^\circ$ ,  $\alpha=6^\circ$  and  $\beta=10^\circ$ . The experiment values were obtained from an integral of the measured pressure distribution. The dotted line shows the local lift distribution at the condition of  $\alpha=6^\circ$ ,  $\beta=0^\circ$  and  $\gamma=0^\circ$  for comparison.

#### 7.5 wing with wing tip tank

The calculation was performed for the wing mentioned in paragraph 7.3 with the ellipsoid of revolution for the wing tip tank. The ratio of length and the maximum diameter of the ellipsoid of revolution is 5 and its length is 320mm which is 2 times wing chord. Fig. 11a and b show the velocity distribution and the pressure distribution obtained from this calculation method. Fig. 12 shows the local lift distributions of the wing with wing tip tank and without wing tip tank under the condition of  $\alpha=4^\circ$ ,  $\beta=0^\circ$  and  $\gamma=10^\circ$ . As can be seen from this figure, the wing with wing tip tank has less pressure loss at the wing tip region and the local lift is large near the wing tip tank. The total number of panels of the model with the wing tip tank was 344.

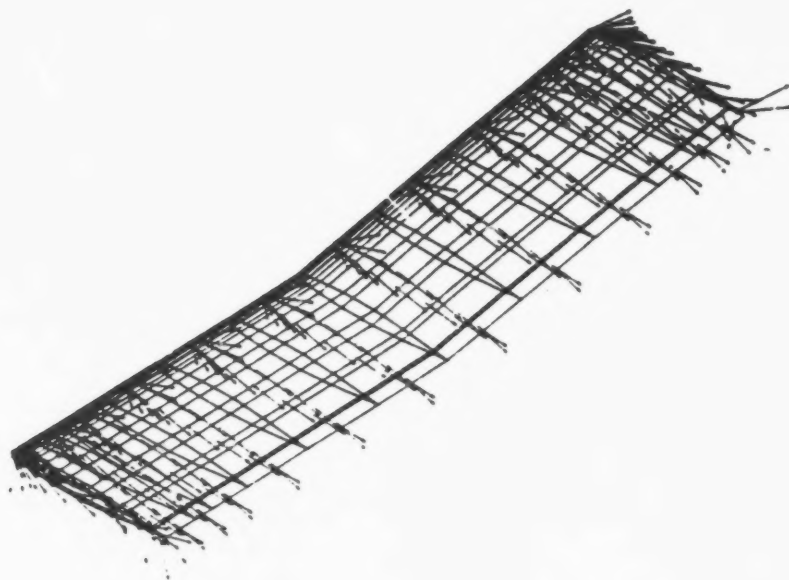


Fig. 8a Velocity vector at the condition of aspect ratio 5, dihedral  $10^\circ$ , angle of attack  $6^\circ$  and side slip angle  $10^\circ$

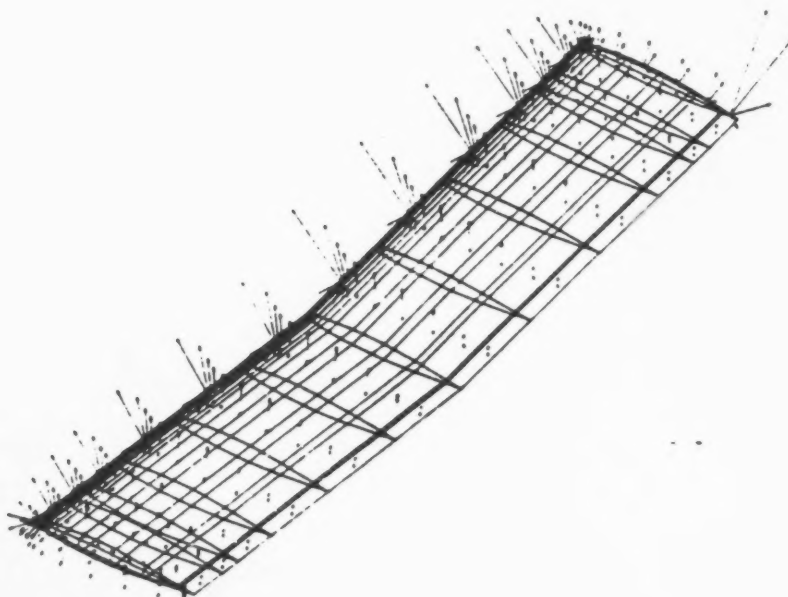


Fig. 8b Pressure distribution at the same conditions as above

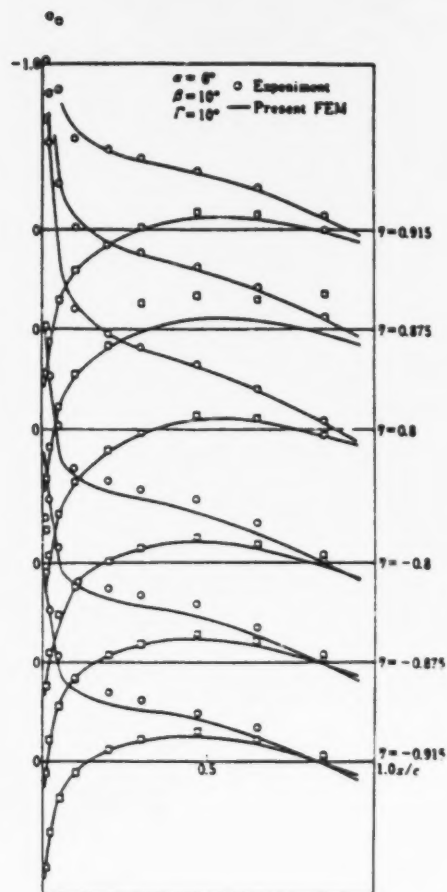


Fig. 9 Comparisons with the experimental values at the same conditions

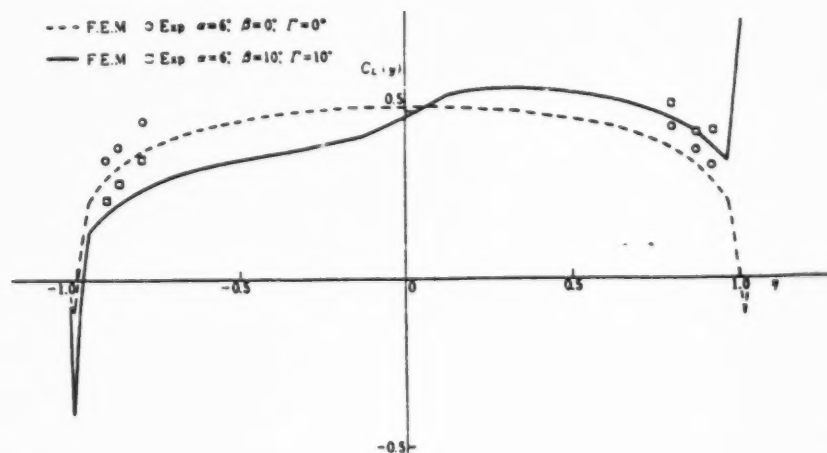


Fig. 10 Local lift distribution at  $\alpha = 6^\circ$

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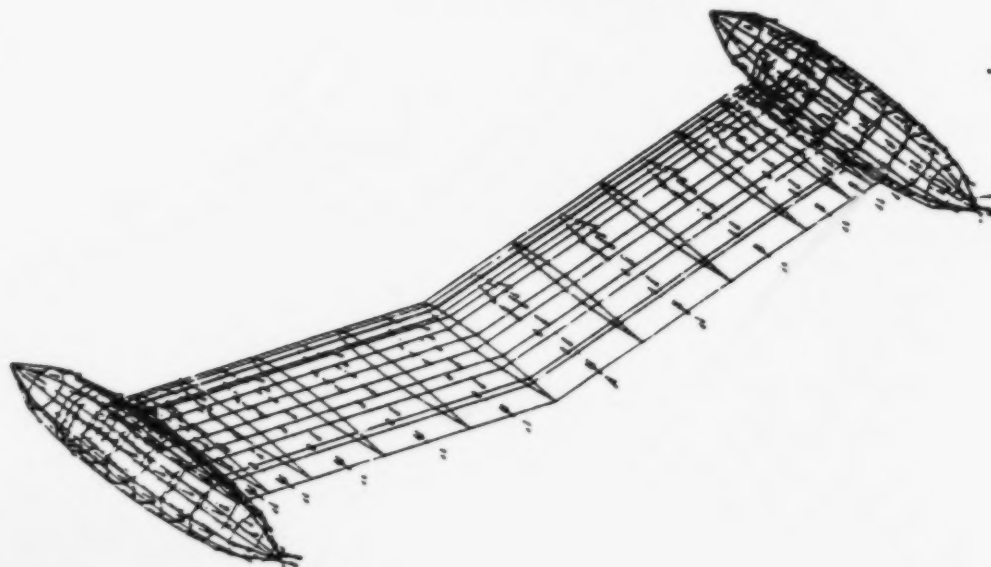


Fig. 11a Velocity distribution of wing with wing tip tank at the conditions of  $\alpha = 4^\circ$ ,  $\beta = 0^\circ$  and  $\Gamma = 10^\circ$

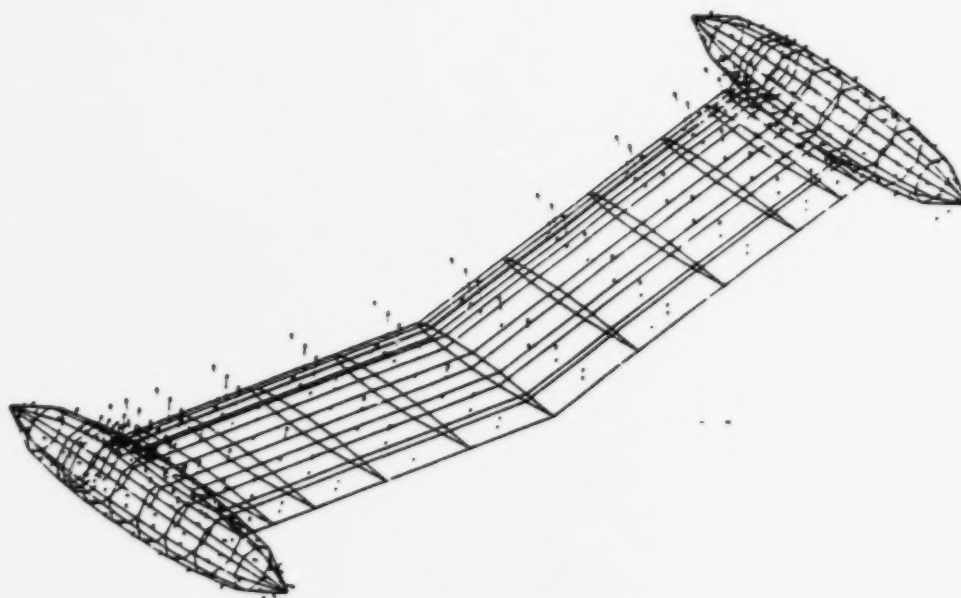


Fig. 11b Pressure distribution at the same condition as above

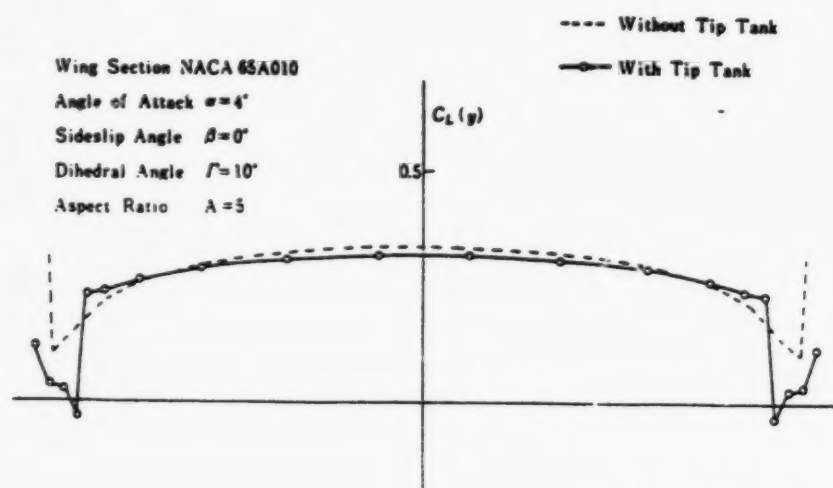


Fig. 12 Local lift distribution of wing with wing tip tank

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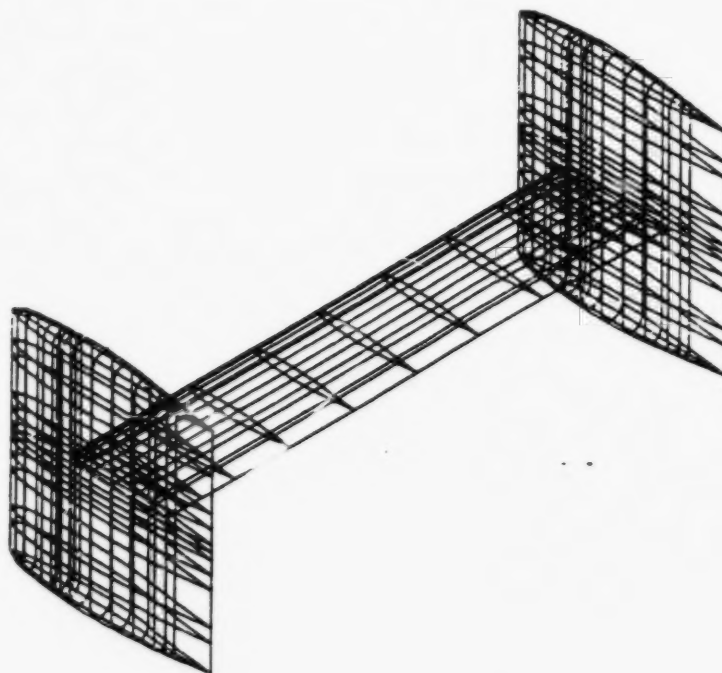
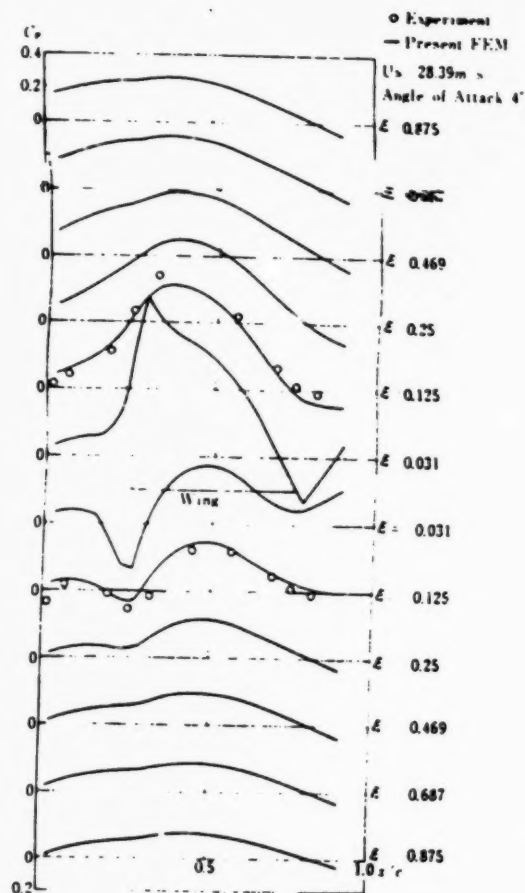


Fig. 13 Wing model with wing tip fence

## 7.6 Wing with wing tip fence

/13

This model has the wing mentioned in paragraph 7.3 with the wing tip fence. The chord length of the wing tip fence is 320mm which is two times larger than that of the wing and its aspect ratio is 1. The aerofoil is NACA65A010 which is the same as the aerofoil of the wing. The wing is attached to the center of the wing tip fence with the incidence  $0^\circ$ . Fig. 13. The model has 10 measuring points at the inside surface of the wing tip fence along the lines of 20mm above and below the wing chord line. Fig. 14b shows the pressure distribution on the wing surface obtained from the calculation and from the experiment in the case of  $4^\circ$  of angle of attack. Fig. 14a shows the pressure distribution on the wing tip fence. The dotted line of Fig. 14b shows the pressure distribution of the wing with the same aspect ratio and it indicates that lift is produced even near the wing tip due to the effects of the wing tip fence. The total number of panels of this wing model with wing tip fence is 1000.



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Fig. 14a Pressure distribution inside of the wing tip fence at  $\alpha = 4^\circ$

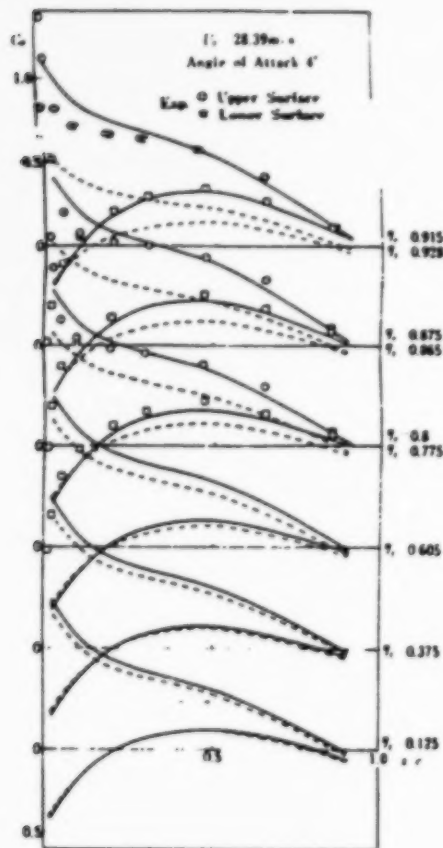


Fig. 14b Pressure distribution of the upper and lower surface of the wing with wing tip fence

#### 7.7 Wing and fuselage joint model

The authors have reported for mid-wing and low-wing in Reference 11 and 13. In this report, an example of computation results of low-wing is given. Fig. 15a and 15b show the velocity distribution and pressure distribution in the case of the angle of attack  $\alpha = -4^\circ$ . If this figure is looked at up side down, it is equivalent to the case of high-wing at the angle of attack  $\alpha = 4^\circ$ . The total number of panels of this low-wing model is 690. Fig. 16a shows the pressure distribution on the fuselage at the angle of attack  $\alpha = -4^\circ$  and Fig. 16b shows the pressure distribution on the wing. The emission line of the trailing vortex on the fuselage was assumed to be along the meridian on the fuselage which is on the extension of the trailing edge of the fairing and calculated. Reference 11 should be referred to for details.

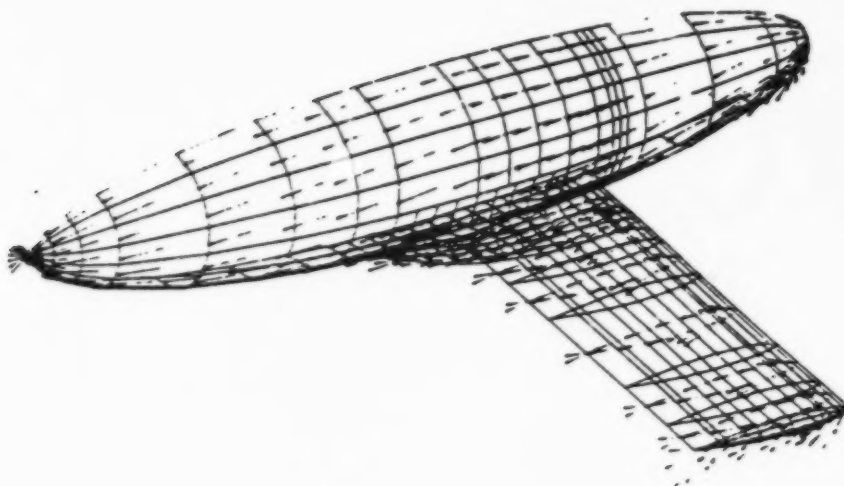


Fig. 15a Velocity distribution of a low-wing model at  $\alpha = -4^\circ$

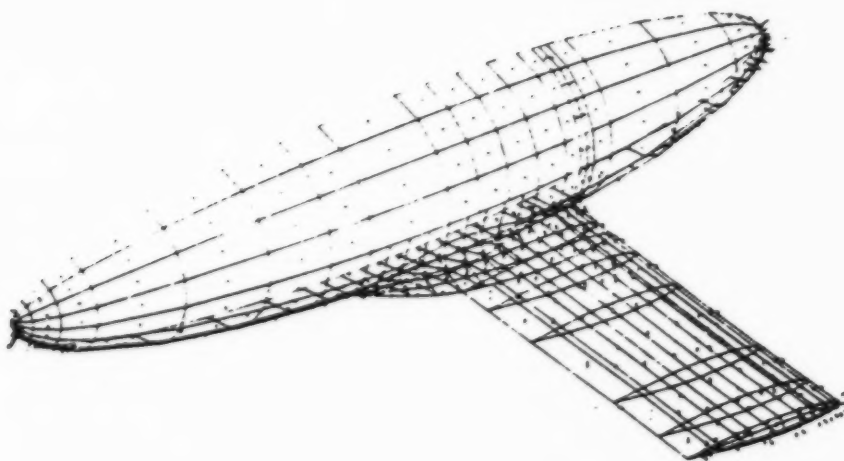


Fig. 15b Pressure distribution of low-wing model at  $\alpha = -4^\circ$

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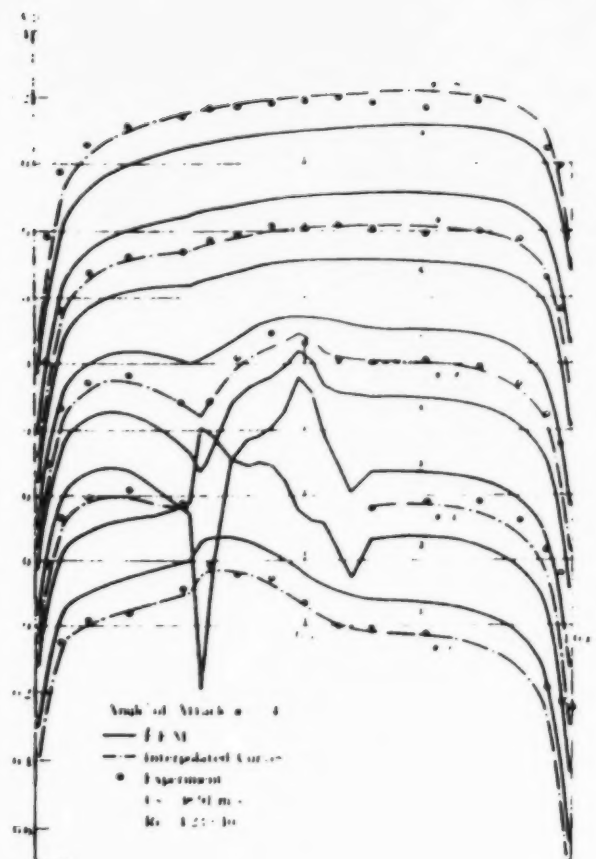


Fig. 16a Pressure distribution on fuselage of low-wing model

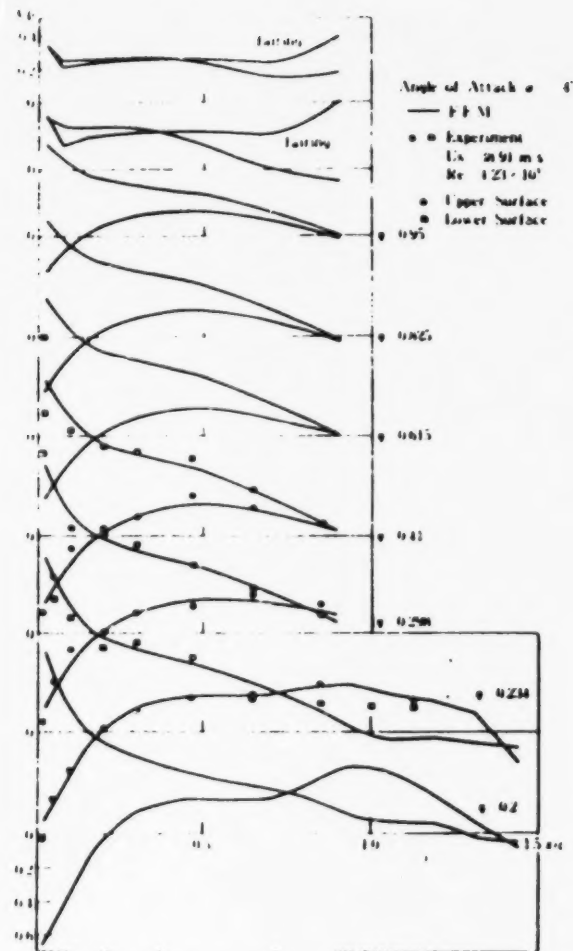


Fig. 16b Pressure distribution on fairing and wing of low-wing model

### 7.8 Tandem wings

/16

The tandem wing has an auxiliary wing behind the wing. In this report, they are called main wing and auxiliary wing. In this calculation method, the trailing vortex plane is always a problem for obtaining pressure distribution of the object creating lift. In the configuration like tandem wings, it is difficult to make a proper assumption of the location of the trailing vortex plane because the auxiliary wing is in the back wash of the main wing. In this calculation, the assumption was made that the trailing vortex plane from the main wing is higher than the auxiliary wing as shown in Fig. 18. The angle of attack of the main wing was set at  $0^\circ$  and 80 cases of the experiment and the calculations were performed using the angle of attack of the auxiliary wing and the relative location of the main and auxiliary wings as the parameters. An example is shown in Fig. 19. This example is the case of the distance between the main wing and the auxiliary wing  $D_x = 20\text{mm}$ , the distance between the wings in the vertical direction  $D_z = 0\text{mm}$ , the angle of attack of the auxiliary wing  $\alpha_2 = 10^\circ$ .

The other cases can be calculated with the accuracy of the case shown in the figure. Reference 12 should be referred to for details.

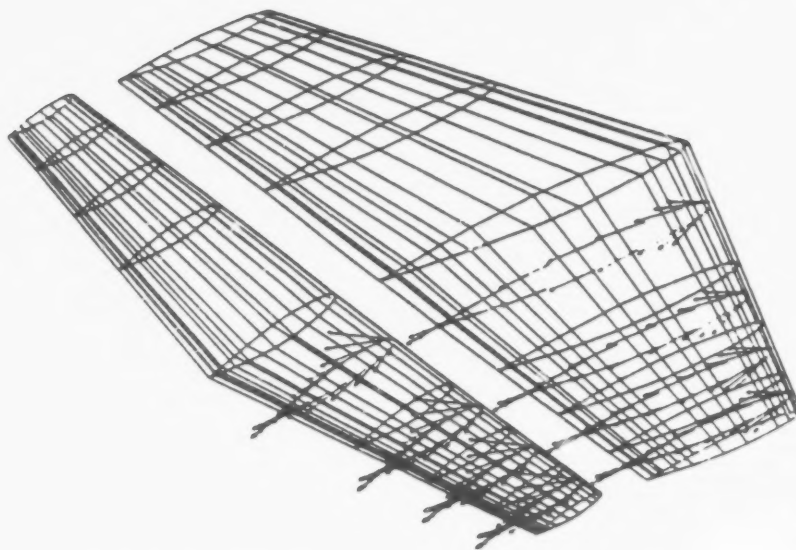


Fig. 17a Velocity distribution at  $\alpha_1 = 0^\circ$ ,  $\alpha_2 = 10^\circ$ ,  $D_x = 40\text{mm}$ .

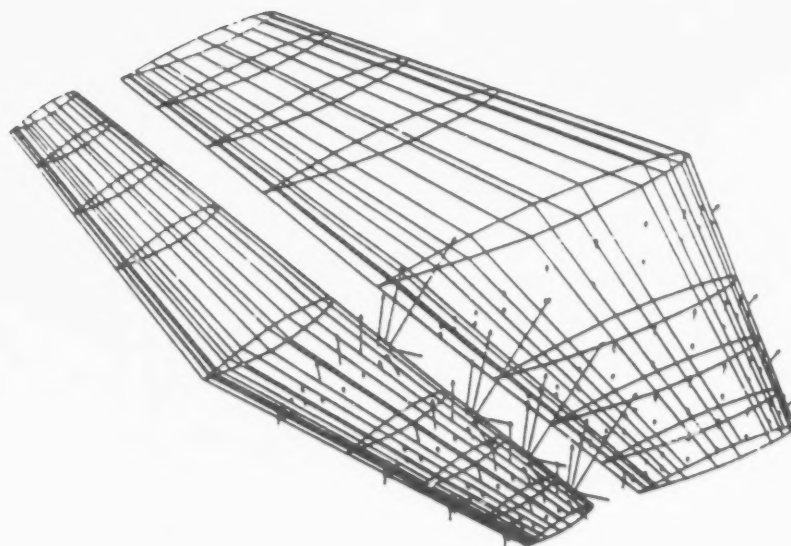


Fig. 17b Pressure distribution at the same conditions as above

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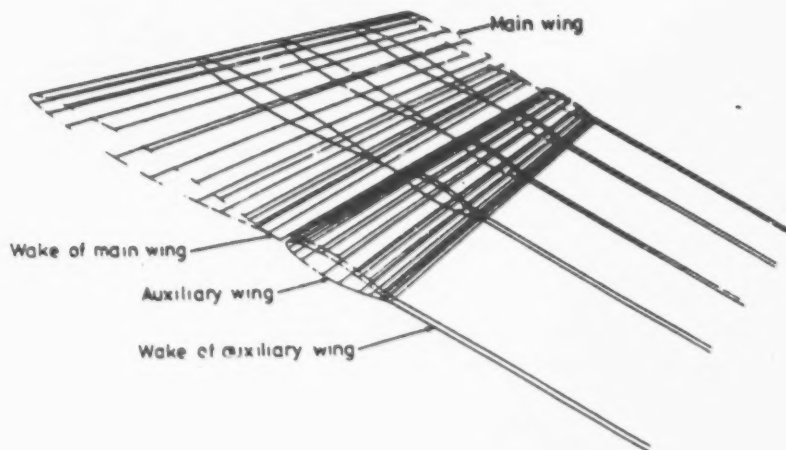


Fig. 18 Assumption of form of trailing vortex plane of wings

#### 7.9 Ground effects

The calculation examples of the models mentioned up to paragraph 7.8 are for models which are surrounded by the panels of the model itself i.e., the case that the difference between the inside and outside of model is clear. Fig. 20. However, this method can be applied to cases where one side of the panel is an open space such as ground effects. Fig. 21 shows the pressure distribution of ground effects in the case where a low wing aircraft flies at a  $4^\circ$  angle of attack and the altitude, which is dimensionless due to the ratio of the altitude and span,  $h = 0.375$ . Fig. 22 shows the lift coefficient for the non-dimensional altitude. Increase of lift coefficient at low altitude by ground effects can be seen.

#### 7.10 Interference of wind tunnel wall

In a similar manner as the ground effects of the previous paragraph, interference of the wind tunnel wall can be calculated. Fig. 23 and Fig. 24 show the pressure distribution of the wind tunnel wall in the case that a sphere is set at the center of the cross section of the wind tunnel whose cross section is a square and one side is 3 times the diameter  $D$  of the sphere. The total number of panels is 960.

The calculations for the 7 models mentioned above were performed. The computer used for the calculations is the FACOM 230-75 model by Fujitsu K.K. The computing time is not important due to rapid advances of computers but the computing time of an example of 690 of panels of low-wing aircraft was 409 seconds. In general, the computing time increases in proportion to the square of the number of panels.

## 8. Conclusions

Paragraph 7 showed the calculations of the various models using the developed program and also showed the plots whenever the experiment data is available. The pressure distributions at low speed of the experiment data agree with the calculation as long as flow separation does not occur. However, a problem for the assumption of location of the trailing vortex emission line remains. The emission line for the model having an airfoil shape should be at the trailing edge but for ellipsoid shape models, the flow separation line has to be obtained from viscous fluid. In the case that it is solved with inviscid fluid like the calculation method in this report, a proper assumption is necessary.

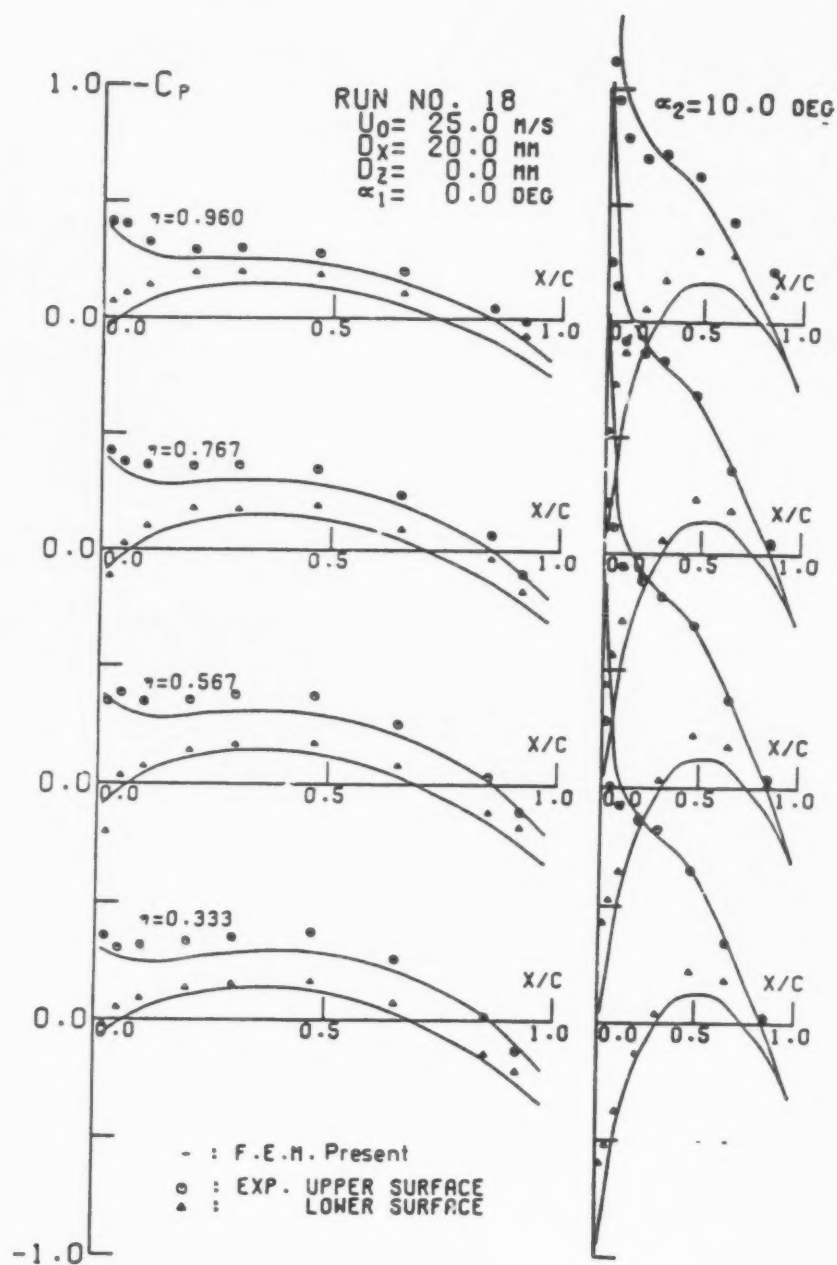


Fig. 19 Pressure distribution at  $\alpha_1 = 0^\circ$ ,  $\alpha_2 = 10^\circ$  and  $D_x = 20\text{mm}$ .

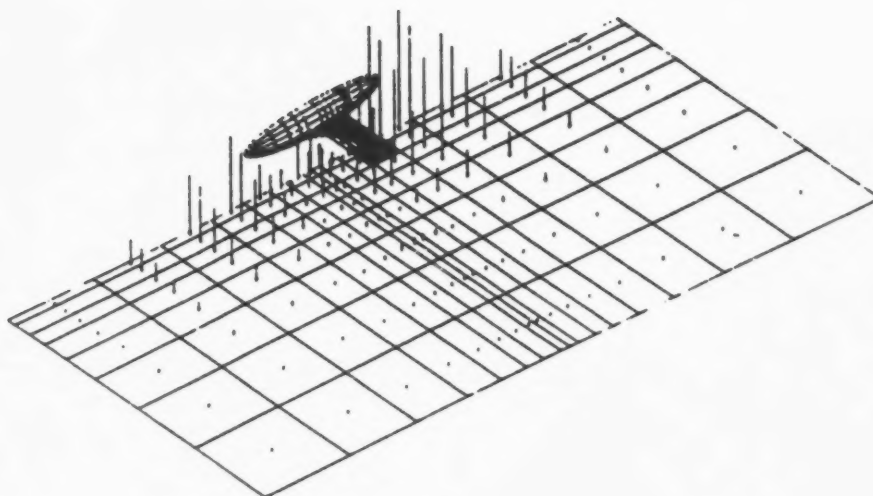


Fig. 20 Pressure distribution at nondimensional altitude  $h = 0.375$  and  $\alpha = 4^\circ$

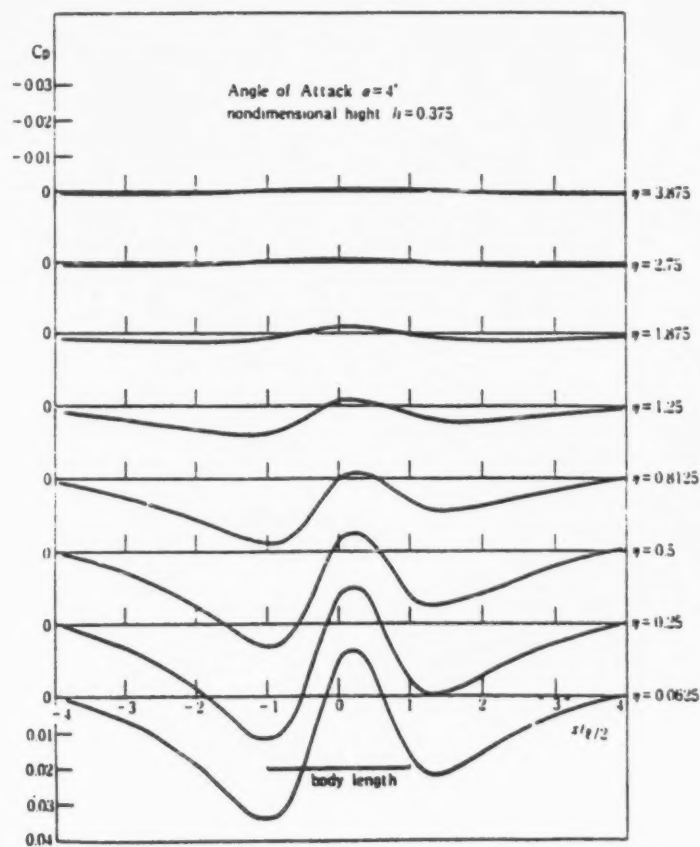


Fig. 21 Pressure distribution at  $y$  which is made nondimensional by the half span.

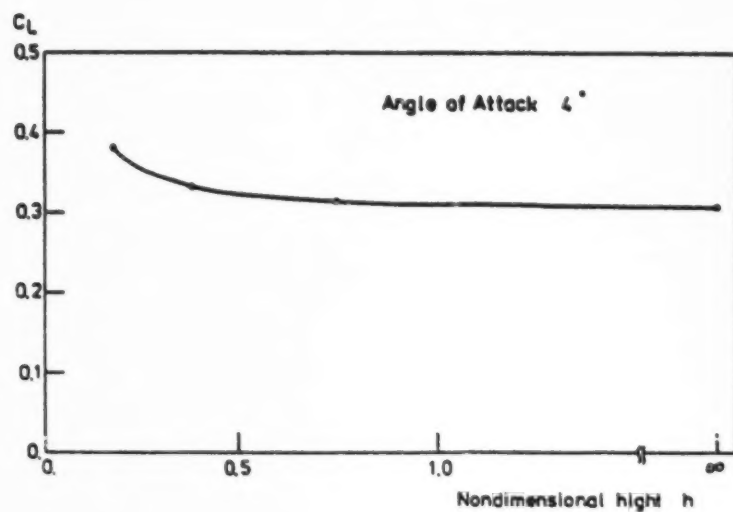


Fig. 22 Lift increase by nondimensional altitude decrease

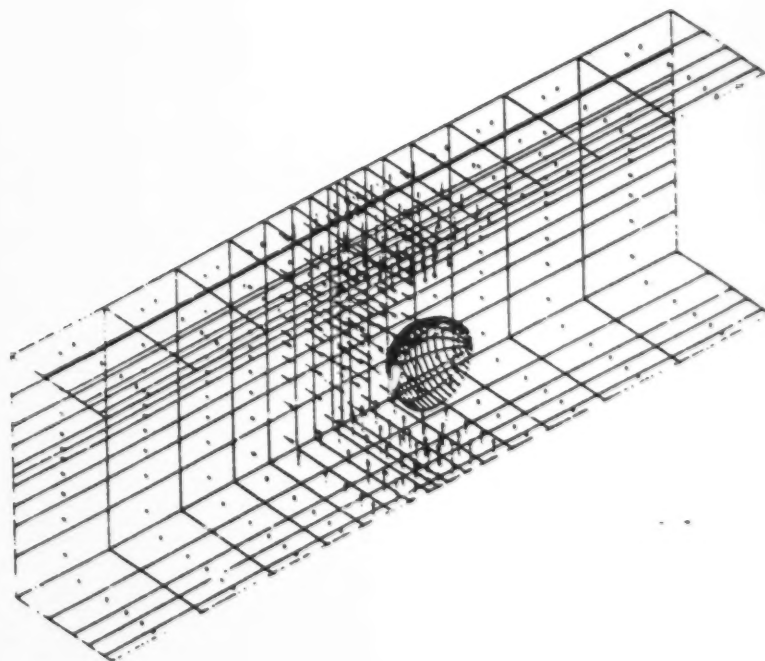


Fig. 23 Pressure distribution of wind tunnel wall in the case that a sphere is set in the center of the wind tunnel

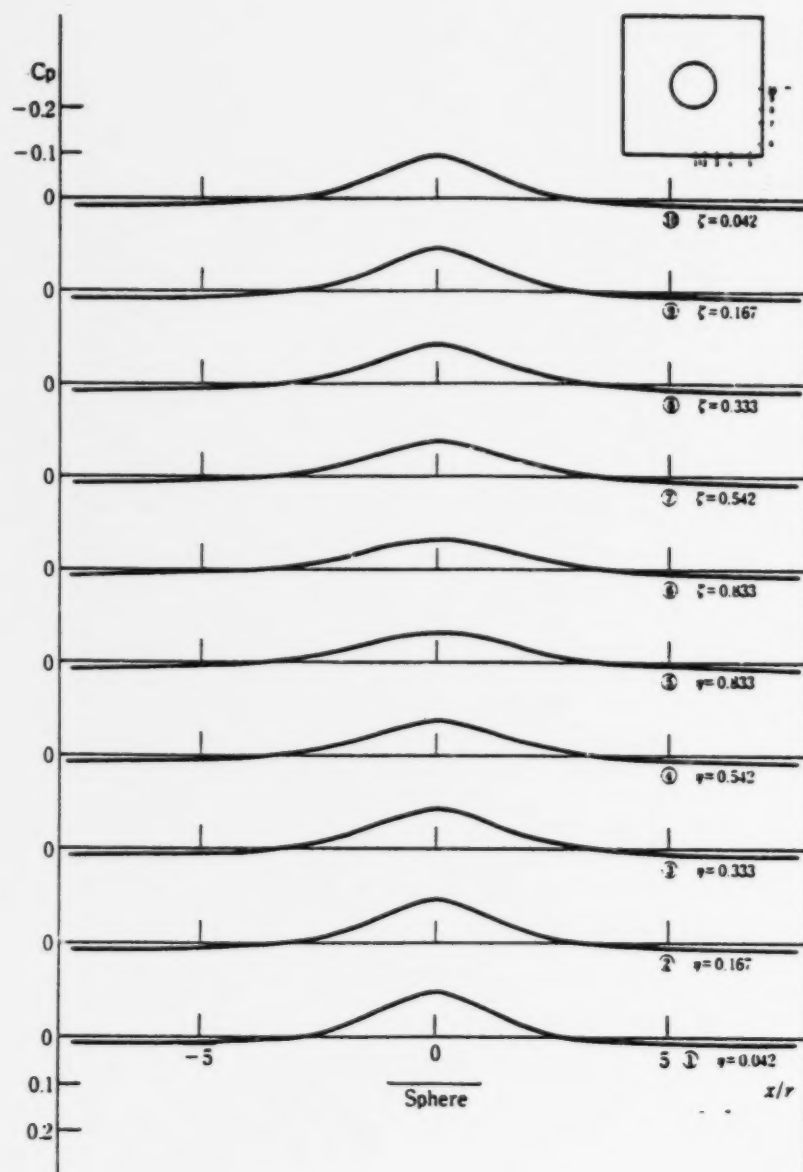


Fig. 24 Pressure distribution of the square wall of the wind tunnel

By comparing this method with the differential method in Reference 7, this method /22  
can calculate the pressure of all of the panels and the area of each element can  
be obtained clearly since the pressure at the center of the panel is obtained.  
Therefore, obtaining accurate entire aerodynamic coefficients was proved from  
the models with various configurations due to an accurate area which is the base in  
the case of calculation of lift, moment and etc.

Recently, Reference 10 by Morino has been published. It is the latest report using  
Green's function. Volume I discusses the theory and the actual program which will  
be in volume II will be published in the near future.

The authors cordially thank the late Hisaichi Washizu, honorary professor of Tokyo  
University who guided this entire study and Akira Azuma, professor of Tokyo Univer-  
sity and his staff for the experiment of the low-wing aircraft model. Mr. Hiromatsu  
who is a leader of the new aircraft research group and his staff helped for the  
other experiments and Mr. Toda who is chief of the aerodynamic group and his staff  
also cooperated in this study.

#### References

- 1) Hess, J. H. and Smith, A. M. O.; Calcula-  
tion of Potential Flow about Arbitrary  
Bodies, Progress in Aeronautical Science,  
Vol. 8, 1967.
- 2) Rubber, P. E. and Saaris, G. R.; Review  
and Evaluation of a Three-Dimensional  
Lifting Potential Flow Analysis Method for  
Arbitrary Configurations, AIAA Paper No.  
72-188, 1972.
- 3) Heckmann, G. et Vitte, W.; Écoulement à  
Potential Tridimensionnel Portant, l'Aéro-  
nautique et l'Astronautique, No. 35, Mars.  
1972.
- 4) Woodward, F. A.: An Improved Method  
for the Aerodynamic Analysis of Wing-  
Body-Tail Configurations in Subsonic and  
Supersonic Flow  
Part I Theory and Application  
Part II Computer Program Description  
NASA CR-2228 May, 1973

- 5) Miranda, L. R.; Extended Applications of the Vortex Lattice Method, NASA SP-405, May 1976.
- 6) Morino, L., Chen, L. T. and Suciu, E. O.; Steady and Oscillating Subsonic and Supersonic Aerodynamics around Complex Configurations, AIAA Journal, Vol. 13, No. 3, March 1975.
- 7) Suzuki, S. and Washizu, K.; Calculation of Wing-Body Pressures in Incompressible Flow Using Green's Function Method. J. Aircraft, Vol. 17, No. 5, May 1980.
- 8) Lamb; Hydrodynamics, Cambridge University Press, 1932.
- 9) Shigefumi Tatsumi : Kokuki no Kukirikigakuteki Mondai eno Yugen Yosoho no Oyo, The engineering department of Tokyo University, master's thesis, March, 1981.
- 10) Morino, L.; Steady, Oscillatory and Unsteady Subsonic and Supersonic Aerodynamics - Production Version (SOUSSA-P1, 1) - Volume I - Theoretical Manual, NASA CR-159130, Jan. 1980.
- 11) Mitsunori Yanagizawa, Kazuo Kikuchi : Yokudo Ketsugotai no Kuriki Tokusei ni kansuru Kenkyu, Dai 2 ho, Teiyokuki ni tsuite, Dai 10 ki, Nihon Koku Uchu Gakkai, April, 1979
- 12) Mitsunori Yanagizawa, Kazuo Kikuchi : Tandem Yoku no Shuyoku to Sodayoku no Kansho Tokusei, Dai 12 ki, Nihon Koku Uchu Gakkai, April, 1981
- 13) Mitsunori Yanagizawa, Kazuo Kikuchi : Yokudo Ketsugotai no Kukirikigakuteki Tokusei ni kansuru Kenkyu, Dai 1 po, Chuyokuki ni tsuite, Kogiken Hokoku TR-729, August, 1982
- 14) Hayato Togawa, Matrix no suchi Keisan. Ohm Sha, 1971
- 15) Masayoshi Fukuda : Shin AP Sub-routine no Toroku, Sengata Hoteishiki o toku. Kogiken, Keisan Center, Computer Techniques No.2, May, 1979.

## 1. Calculation of aerodynamic influence coefficient (Reference 6)

The position vector  $P$  at an arbitrary point of the element  $\Sigma_k$  is expressed as follows with the local coordinate system  $(\xi, \eta)$  defined in the element.

$$P = P_0 + \xi P_1 + \eta P_2 + \xi \eta P_3 \quad (A1-1)$$

There is a relationship between  $P_i$  ( $i=0$  to 3) and the position vector ( $P_{++}$ ,  $P_{+-}$ ,  $P_{-+}$ ,  $P_{--}$ ) at the 4 corners of a quadrangle.

$$\begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} P_{++} \\ P_{+-} \\ P_{-+} \\ P_{--} \end{pmatrix} \quad (A1-2)$$

The influence coefficient on the element  $\Sigma_k$  is analytically calculated.

$$C_{kk} = I_D(1, 1) - I_D(1, -1) - I_D(-1, 1) + I_D(-1, -1) \quad (A1-3)$$

$$B_{kk} = I_S(1, 1) - I_S(1, -1) - I_S(-1, 1) + I_S(-1, -1) \quad (A1-4)$$

$$I_D(\xi, \eta) = \frac{1}{2\pi} \tan^{-1} \left[ \frac{R \times a_1 \cdot R \times a_2}{|R| \cdot (R \cdot a_1 \times a_2)} \right] \quad (A1-5)$$

$$I_S(\xi, \eta) = \frac{1}{2\pi} \left\{ -R \times a_1 \cdot n \frac{1}{|a_1|} \sinh^{-1} \left( \frac{R \cdot a_1}{|R \times a_1|} \right) + R \times a_2 \cdot n \frac{1}{|a_2|} \sinh^{-1} \left( \frac{R \cdot a_2}{|R \times a_2|} \right) + R \cdot n \tan^{-1} \left( \frac{R \times a_1 \cdot R \times a_2}{|R| \cdot (R \cdot a_1 \times a_2)} \right) \right\} \quad (A1-6)$$

where  $\tan^{-1}$  has the principal value between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$ .

$$R(\xi, \eta) = P - P_k = P_0 + \xi P_1 + \eta P_2 + \xi \eta P_3 - P_k$$

$$a_1(\xi, \eta) = \frac{\partial R}{\partial \xi} = P_1 + \eta P_3$$

$$a_2(\xi, \eta) = \frac{\partial R}{\partial \eta} = P_2 + \xi P_3$$

$$n(\xi, \eta) = \frac{a_1 \times a_2}{|a_1 \times a_2|}$$

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$W_{hk}$  can be calculated in a similar manner as  $C_{hk}$ .

## Supplement 2

### 2. Symmetry object

The degree of freedom can be  $\frac{1}{2}$  due to the symmetric property when the model is divided and discretized. Thus equation (7) can be written as follows.

$$\sum_{k=1}^K (\delta_{kk} - C_{kk} - W_{kk}) \cdot \varphi_k = B_k \quad (A2-1)$$

In the case that the model is symmetrical about the plane  $y=0$ , the equation (A2-1) is

$$\sum_{k=1}^{\hat{N}} (\delta_{kk} - \hat{C}_{kk} - \hat{W}_{kk}) \cdot \varphi_k = B_k \quad (A2-2)$$

where  $\hat{N} = \frac{N}{2}$  which is the number of the right side panels of the model. Since  $\hat{C}_{hk}$  is symmetry about the plane  $y = 0$ , the aerodynamic influence coefficients of the right side and the left side are added.

$$\begin{aligned} \hat{C}_{kk} = & \frac{1}{2\pi} \iint_{\Sigma_1^{(k)}} \left[ \frac{\partial}{\partial n} \left( \frac{1}{R} \right) \right]_{n1} ds \\ & + \frac{1}{2\pi} \iint_{\Sigma_1^{(L)}} \left[ \frac{\partial}{\partial n} \left( \frac{1}{R} \right) \right]_{(L)1} ds \end{aligned} \quad (A2-3)$$

where

$$\begin{aligned} R^{(H)} &= \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \\ R^{(L)} &= \sqrt{(x-x_0)^2 + (y+y_0)^2 + (z-z_0)^2} \end{aligned} \quad (A2-4)$$

$\hat{W}_{hk}$  can be calculated in a similar manner.

## Supplement 3

### 3. Trailing vortex plane

The vorticity zone occurs in the back wash of a three dimensional object creating lift.

If the flow is assumed to be continuous and an incompressible fluid, the following equation is obtained along the stream line according to Bernoulli's theorem.

$$\frac{1}{2} \mathbf{U} \cdot \mathbf{U} + \frac{P}{\rho} = \frac{1}{2} \mathbf{U}_\infty \cdot \mathbf{U}_\infty + \frac{P_\infty}{\rho} \quad (A3-1)$$

where  $\mathbf{U}$  is the velocity vector,  $\mathbf{U}_\infty$  is the velocity of the uniform flow, the vector  $\varphi$  is the disturbance velocity potential.

$$\mathbf{U} = \mathbf{U}_\infty + \nabla \varphi \quad (A3-2)$$

Since the static pressure at the upper and the lower planes at the arbitrary point in the trailing vortex plane is equal, the following equation is obtained by using equations (A3-1) and (A3-2).

$$\begin{aligned}
 & | (U_\infty + v\varphi) \cdot (U_\infty + v\varphi) |_{upper} \\
 & = | (U_\infty + v\varphi) \cdot (U_\infty + v\varphi) |_{lower} \\
 & \quad (A3-3)
 \end{aligned}$$

If the term having a higher order than the square term is neglected,

$$v(\varphi_{upper} - \varphi_{lower}) = 0 \quad (A3-4)$$

This indicates that the difference of the disturbance velocity potential  $\Delta\varphi$  along the stream line in the trailing vortex plane is constant.

#### Supplement 4

When the object which creates lift is panelized, determination of the number of divisions is always a problem. It is especially necessary to divide it into small panels near the leading edge of the wing. Generally, the region where pressure changes rapidly has to be divided into small panels but the region where pressure change is small can be divided with large panels. Fig. A4-1 shows the calculation results of the case that the model was divided into especially small panels chordwise rather than spanwise.

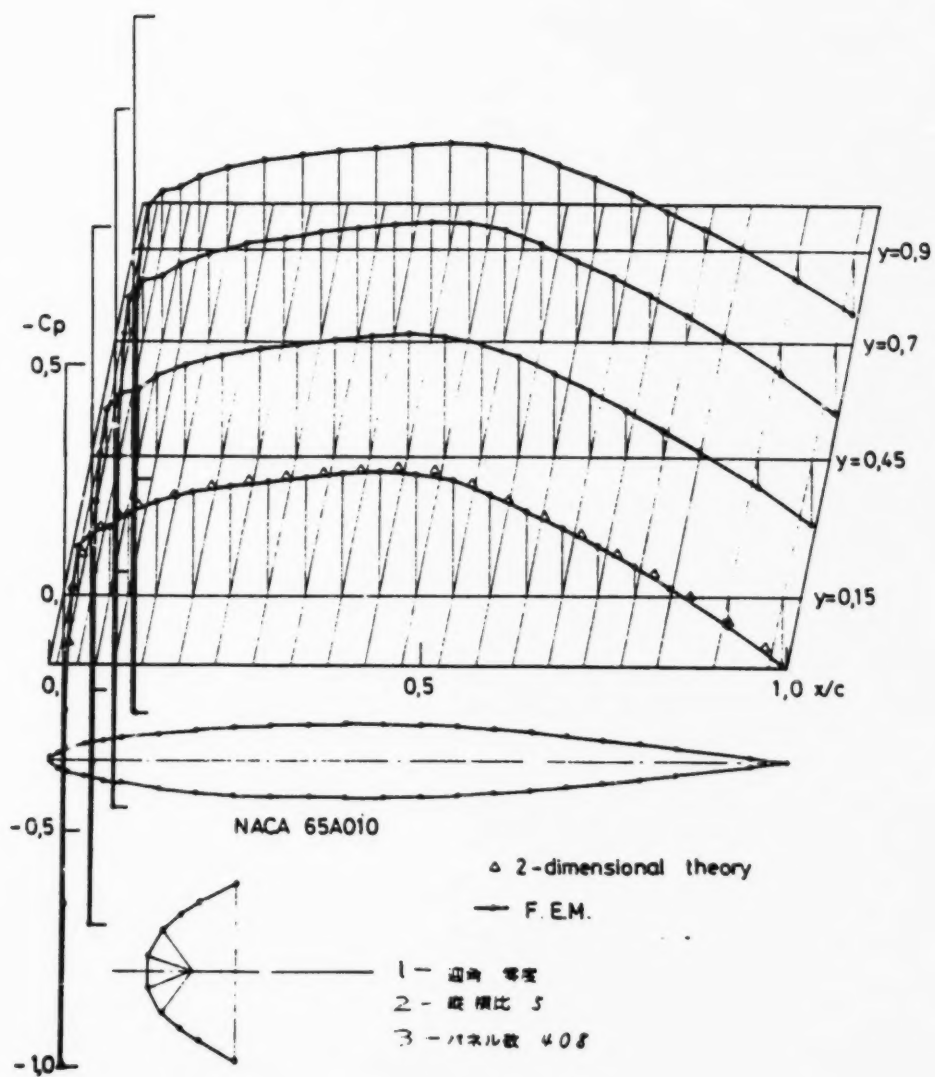


Fig. A4-1 Pressure distribution of the wing divided into small panels,  $\alpha = 0^\circ$   
 1- Angle of attack  $0^\circ$   
 2- Aspect ratio 5  
 3- Number of panels 408

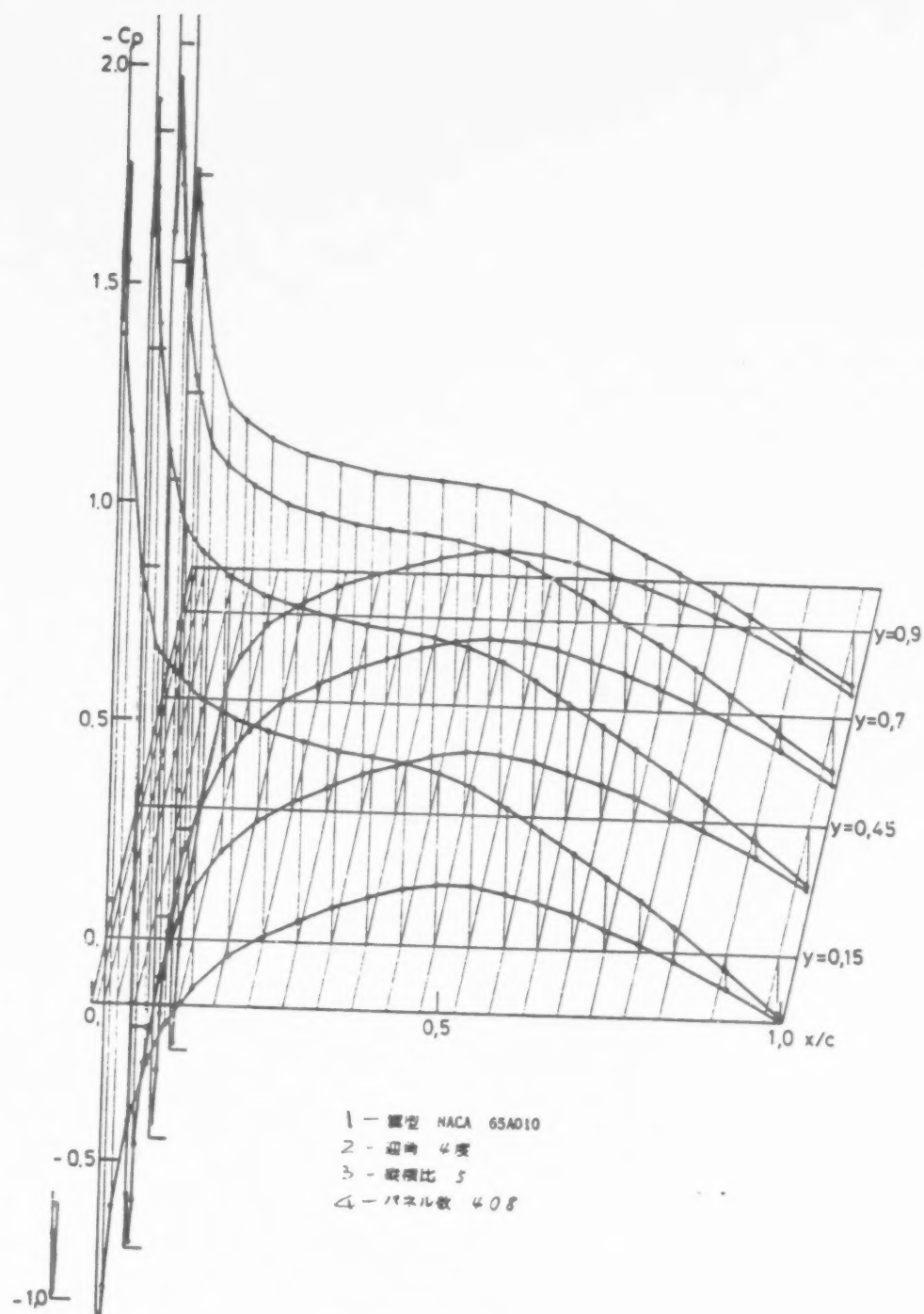


Fig. A4-2 Pressure distribution of the wing divided into small panels,  $\alpha = 4^\circ$

1- Aerofoil NACA65A010

2- Angle of attack  $4^\circ$

3- Aspect ratio 5

4- Number of panels 408

NC = 51 and in this case, it is the number of panels of the upper and lower planes. /26  
 As shown in Fig. A4-1, the aerofoil is NACA65A010 and the figure shows the 9 divisions at the leading edge region. The tip panel faces  $90^\circ$  against the flow. When the angle of attack  $\alpha$  is  $0^\circ$ , the pressure coefficient  $C_p$  at the stagnation point of the panel is calculated 1.0. The triangle  $\Delta$  plots compared with the two dimensional theoretical values. Fig. A4-2 shows the pressure distribution in the case of the angle of attack of  $4^\circ$  and when the leading edge region is divided into small panels, it can be noticed that the pressure distribution at the upper and lower surface is continuous. Also it is noticed that the stagnation point moves one panel below. For the three dimensional object, increase of number of panels at the leading edge of the wing causes increase of the total number of panels so usually the number of panels is compromised with an adequate number.

#### Supplement 5

#### 5. Solution of simultaneous linear equations using external memory system

The FACOM 230-75 model of Fujitsu K.K. was used for the calculations in this report. Its maximum core memory which the user can use is approximately 750KWORD. The maximum number of elements required to solve the simultaneous linear equation of the finite element method within the limit of the core memory is about 800. Thus an external memory system is required to solve the problem with more than the number of elements. However the method published requires complicated operations of input and output so the channel time increases significantly. Thus the authors prepared a program to suppress the increase of the channel time and to solve large simultaneous linear equations by the block elimination method (Reference 14).

#### (1) Computing method

In the block elimination method (Reference 14), the coefficient matrix  $[A]$  is divided adequately and the divided matrix is operated, i.e., eliminated, as an element of the matrix. For this elimination, the Gaussian or the Gauss-Jordan method was used.

The original equation is

$$A_{j_0, m_0} \{ x_{j_0} \} = \{ B_{j_0} \} \quad (A5-1)$$

where  $t_0, j_0$  is 1 to  $m_0$  and  $m_0 = m_1 \cdot m_2$  (The equation A5-1 is an equation which expresses the entire number of elements but this is divided into minor matrices and rearranged for easy calculation.)

$$\begin{aligned}
 (A_{i1,j1}) &= (A_{i0,j0}) & (5-2) \\
 (\varphi_{j1}) &= (\varphi_{j0}) \\
 (B_{j1}) &= (B_{j0})
 \end{aligned}$$

where  $i1, j1 = 1$  to  $m2$

$$\begin{aligned}
 i0 &= (i1 - 1) \cdot m1 + 1 \sim i1 \cdot m1 \\
 j0 &= (j1 - 1) \cdot m1 + 1 \sim j1 \cdot m1
 \end{aligned}$$

Fig. A5-1 explains this operation. For instance, if the original matrix has elements of  $1000 \times 1000$ , it is divided to be  $10 \times 10$  and the figure shows the process to make the minor matrices. The number of elements of  $[A_{i0,j0}]$  and  $[a_{i1,j1}]$  is the same but it is in order of from top to bottom. The first row of the block elimination method is explained as follows.

$$\begin{aligned}
 & \begin{pmatrix} A_{11}^{-1} & 0 & 0 & \cdots & 0 \\ -A_{21} \cdot A_{11}^{-1} & E & 0 & \cdots & 0 \\ -A_{31} \cdot A_{11}^{-1} & 0 & E & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_{m21} \cdot A_{11}^{-1} & 0 & 0 & \cdots & E \end{pmatrix} \\
 & \times \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1,m2} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2,m2} \\ A_{31} & A_{32} & A_{33} & \cdots & A_{3,m2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{m2,1} & A_{m2,2} & A_{m2,3} & \cdots & A_{m2,m2} \end{pmatrix} \\
 & = \begin{pmatrix} E & A_{12}^1 & A_{13}^1 & \cdots & A_{1,m2}^1 \\ 0 & A_{22}^1 & A_{23}^1 & \cdots & A_{2,m2}^1 \\ 0 & A_{32}^1 & A_{33}^1 & \cdots & A_{3,m2}^1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & A_{m2,2}^1 & A_{m2,3}^1 & \cdots & A_{m2,m2}^1 \end{pmatrix} \\
 & \quad (A5-3)
 \end{aligned}$$

The equation (A5-3) is written as below for simplicity.

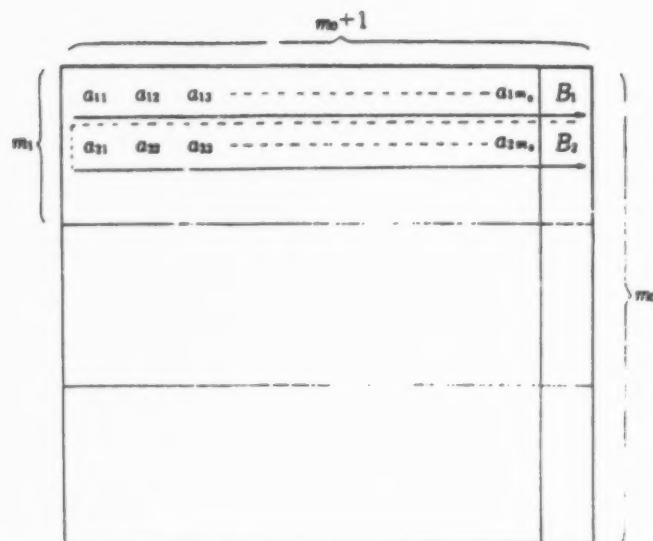
$$(D_{i1,j1}) \times (F_{i1,j1}) = (F_{i1,j1}^1) \quad (A5-4)$$

where  $A_{ij}$  is a minor matrix,  $A_{ij}^{-1}$  is the inverse matrix of  $A_{ij}$ .  $A_{ij}^1$  is the matrix which is from the operation of the first row of  $A_{ij}$ ,  $E$  is identity matrix,  $0$  is zero matrix.

By operating the above process, the elements are eliminated by each block. The elimination of No.  $n$  row can be shown as follows.

$$(D_{11,11}^{(n)}) \times (F_{11,11}^{(n)}) = (F_{11,11}^{(n)}) \quad (A5-5)$$

/27



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Fig. A5-1a Matrix obtained by the finite element method

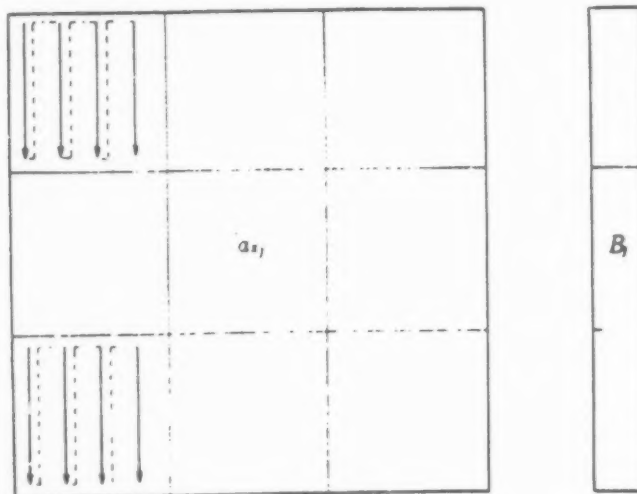


Fig. A5-1b Matrix arranged for easy calculation by dividing minor matrices

$$(D_{11,11}^{(n)}) = \begin{bmatrix} E & \cdots & -A_{1,n}^{(n-1)} & \cdots & A_{1,n}^{(n-1)} & \cdots & 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \cdots & & A_{n,n}^{(n-1)} & \cdots & 0 & \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \cdots & -A_{m,n}^{(n-1)} & \cdots & A_{m,n}^{(n-1)} & \cdots & E \end{bmatrix} \quad (F_{11,11}^{(n)}) = \begin{bmatrix} E & \cdots & 0 & A_{1,n}^{(n)} & \cdots & A_{1,m}^{(n)} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & E & A_{n,n}^{(n)} & \cdots & A_{n,m}^{(n)} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & A_{m,n}^{(n)} & \cdots & A_{m,m}^{(n)} \end{bmatrix}$$

Finally,  $\{F_{i1,n}^{(m)}\} = \{E\}$

i.e.,

$$\{F_{i1,n}^{(m+1)}\} = \{F_{i1,n}^{(m)}\} \times \dots \times \{F_{i1,n}^{(1)}\} \times \{F_{i1,n}^{(0)}\} = \{A_{i1,n}\}^{-1}$$

Thus the solution of the simultaneous linear equation is obtained from

$$\{v_n\} = \{A_{i1,n}\}^{-1} \{B_n\} \quad (A5-6)$$

## (2) Input and output operation

This method involves a large amount of input and output data because the intermediate values in the calculation have to be stored in the file. Thus it takes long time with the ordinary READ/WRITE command so, by using BLOCK READ/WRITE, the CPU time was shortened.

The characteristics of this method are as follows:

- (1) It can be used for the solution when the number of panels exceeds the memory capacity of the main memory system.
- (2) The application limit is large because of extension of the Gauss-Jordan elimination method.
- (3) The coefficient matrix has to be arranged to fit the program.
- (4) In the case that the capacity of the main memory system is large enough for the solution, it is faster to calculate without using this program.

When the number of panels is small, the subroutine by APU described in Reference 15 was used if the number of panels is less than about 600. The calculation by APU was used only for the solution of the linear equation by VSWEPR registered in NALAPLIB.

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